**Forecast Models:** quarterly data only.

Here a little summary of what I have done. I have constructed a lot of different VARs for Brazil based on only the variables of Pablo that are quarterly available. Later on I will mix them up with the monthly data. The only purpose here is to get an idea of the forecast accuracy of the models. Before we can make forecast we have to construct each VAR model and I stuck to the following steps.

**Step 1:** *Data selection N-variable kth-order reduced-form*

Of course, the first step would be to decide on which variables to include in and on the frequency of the data. I decided to first experiment with the quarterly data and focus on 4th-order VARs. So VARs with 4 variables. For the 4th-order VARs that performed well in forecasting, I expanded them to 5 or 6 variables to see how that would affect the forecast accuracy.

**Step 2:** Test for stationarity: Augmented Dickey-Fuller (ADF) test.

After we decided on the N-variables to include in the VAR we should run an ADF test on each variable in order to determine whether the variables are a stationary I(0) process or a non-stationary I(1) process. A notion must be made that the ADF-test has one significant disadvantage. That is, it models the time-series either as a full non-stationary unit root process or as a full stationary process.

All variables are non-stationary except for the variable *primario*. Therefore, I have chosen to construct a VAR in first differences.

**Step 3:** Number of lags to include in the VAR

After we have decided on the *N-*variables to include and dealt we the stationarity issues, we should determine the number of *k-*lags to include. For this purpose we should do a lag-test. In stata you have the option *varsoc* that will give you various lag tests. I have used this option and decided for each model what the optimal lag structure would be.

**Step 4:** Serial Autocorrelation in the residuals

Forecast can still be unbiased when there is autocorrelation in the residuals but they become inefficient. Therefore I perform a LM-test to see whether autocorrelation is a problem or not.

**Step 5:** Stability of the VAR

After we have obtained the number of lags we have to examine whether the VAR is stable. This condition is satisfied if and only if all inverse roots of the determinant of the characteristic polynomial have a modulus .

If the VAR is stable we can rewrite the VAR in moving average form and start forecasting.

**Step 6:** granger causality

After all these steps I perform a granger causality test to get an idea and feeling of the dynamics within the model.

**Step 7:** Pseudo-Out-Of-Sample (POOS) Forecasts

Since we are mainly interested in forecast performance of the models I do not spend too much attention to the VAR models themselves. Instead, I thought of a way to judge which model is the best for forecasting. Basically, we have three options. First, we have *in-sample analysis.* In-sample analysis means to estimate the model using all available data up to the latest observation, and then compare the model's fitted values to the actual realizations. However, this procedure is known to draw an overly optimistic picture of the model’s forecasting ability, since common fitting algorithms (e.g. using squared error or likelihood criteria) tend to take pains to avoid large prediction errors, and is thus susceptible to overfitting - mistaking noise for signal in the data.

Second, we could do an *out-of-sample* analysis. A true out-of-sample analysis would be to estimate the model based on data up to the latest quarter, construct a forecast of next quarter’s value, wait until the realization of the next quarter, record the forecast error, re-estimate the model, make a new forecast of the next quarter, and so forth. At the end of this exercise, one would have a sample of forecast errors which would be truly out-of-sample and would give a very realistic picture of the model's performance.

Since this procedure is very time-consuming, forecasters often resort to *"pseudo’’* o*ut-of-sample (POOS) analysis*, which means to mimic the procedure described in the last paragraph, using some historical date, rather than today's date, as a starting point. The resulting forecasting errors are then used to get an estimate of the model's out-of-sample forecasting ability.

So I have decided to do a POOS-analysis to evaluate and identify the VARs that perform the best in forecasting real GDP of Brazil. I have chosen three periods.

**Period 1:** *h*-step ahead forecast from 2010q1 on

**Period 2:** *h*-step ahead forecast from 2012q2 on

**Period 3:** *h*-step ahead forecast from 2013q4 on

Now we still have to decide how we are going to evaluate the forecast accuracy within these sub-samples. I have choosen to use four different measures of forecast accuracy: **1) Root Mean Squared Error (RMSE), 2) Mean Absolute Error (MAE), 3) Correlation between forecasts and realization and 4) Theil’s U.**

I think the four measures are well known to you guys. Maybe Theil’s U is less known. Theil’s U statistic is a relative accuracy measure that compares the forecasted results with the results of forecasting with the naïve model, . It also squares the deviations to give more weight to large errors and to exaggerate errors, which can help eliminate methods with large errors. A value less than means that the forecasts of the VAR are better than guessing, a value of 1 means that the model performs as good as guessing and a value larger than one means that the VAR performs worse than just guessing.

**The Pseudo Out Of Sample (POOS) Forecasts**

Over all models I tried out, I was left with 8 models that seem to forecast the best. To compare these models I construct a graph with the forecasts from the 5 best models and a table of the forecast measures of all 8 models.

**Period 1**

****

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Model** | **Step-ahead** | **RMSE** | **MAE** | **Correlation** | **Theil's U** |
| *VAR7* | *h = 1* | **0.33** | **0.24** | 1.00 | **0.01** |
|  | *h = 2* | 0.86 | **0.63** | 0.98 | 0.07 |
|  | *h = 3* | 0.75 | 0.53 | 0.98 | 0.09 |
|  | *h = 4* | 1.81 | 1.81 | 0.91 | 0.43 |
|  | *h = 5* | 1.99 | 1.43 | 0.91 | 0.59 |
|  | *h = 6* | 1.86 | 1.31 | 0.91 | 0.62 |
|  | *h = 8* | 2.55 | 1.90 | 0.69 | 1.39 |
| *VAR12* | *h = 1* | 2.00 | 1.42 |  | 0.08 |
|  | *h = 2* | 1.67 | 1.15 |  | 0.09 |
|  | *h = 3* | 1.45 | 0.86 |  | 0.08 |
|  | *h = 4* | **1.35** | **0.86** |  | **0.13** |
|  | *h = 5* | **1.23** | **0.73** |  | **0.13** |
|  | *h = 6* | **1.27** | **0.84** |  | 0.56 |
|  | *h = 8* | **1.37** | **1.02** |  | 0.91 |
| *VAR14* | *h = 1* | 0.89 | 0.63 |  | 0.04 |
|  | *h = 2* | 3.70 | 2.51 |  | 0.30 |
|  | *h = 3* | 3.24 | 2.14 |  | 0.39 |
|  | *h = 4* | 3.56 | 2.63 |  | 0.65 |
|  | *h = 5* | 3.40 | 2.60 |  | 0.74 |
|  | *h = 6* | 3.20 | 2.44 |  | 0.91 |
|  | *h = 8* | 4.57 | 3.21 |  | 3.89 |
| *VAR15* | *h = 1* | 2.90 | 2.05 |  | 0.12 |
|  | *h = 2* | 3.07 | 2.49 |  | 0.20 |
|  | *h = 3* | 3.89 | 3.29 |  | 1.50 |
|  | *h = 4* | 3.57 | 2.98 |  | 1.51 |
|  | *h = 5* | 3.29 | 2.68 |  | 1.51 |
|  | *h = 6* | 3.08 | 2.48 |  | 1.58 |
|  | *h = 8* | 3.57 | 2.98 |  | 2.64 |
| *VAR16* | *h = 1* | 2.41 | 1.71 |  | 0.10 |
|  | *h = 2* | 1.98 | 1.27 |  | 0.10 |
|  | *h = 3* | 1.72 | 1.02 |  | 0.12 |
|  | *h = 4* | 1.61 | 1.02 |  | 0.16 |
|  | *h = 5* | 1.48 | 0.92 |  | 0.17 |
|  | *h = 6* | 1.47 | 0.99 |  | **0.54** |
|  | *h = 8* | 1.54 | 1.16 |  | 0.90 |
| *VAR17* | *h = 1* | 2.80 | 1.98 |  | 0.12 |
|  | *h = 2* | 2.30 | 1.50 |  | 0.12 |
|  | *h = 3* | 2.00 | 1.19 |  | 0.13 |
|  | *h = 4* | 1.81 | 1.07 |  | 0.15 |
|  | *h = 5* | 1.71 | 1.07 |  | 0.22 |
|  | *h = 6* | 1.63 | 1.07 |  | 0.45 |
|  | *h = 8* | 1.57 | 1.13 |  | **0.68** |
| *VAR18* | *h = 1* | 0.81 | 0.57 |  | 0.03 |
|  | *h = 2* | **0.81** | **0.65** |  | **0.05** |
|  | *h = 3* | **0.70** | **0.51** |  | **0.06** |
|  | *h = 4* | 1.49 | 1.02 |  | 0.34 |
|  | *h = 5* | 1.58 | 1.18 |  | 0.45 |
|  | *h = 6* | 1.47 | 1.06 |  | 0.47 |
|  | *h = 8* | 2.14 | 1.59 |  | 1.15 |
| *VAR19* | *h = 1* | 1.97 | 1.40 |  | 0.08 |
|  | *h = 2* | 2.05 | 1.66 |  | 0.13 |
|  | *h = 3* | 2.76 | 2.30 |  | 1.11 |
|  | *h = 4* | 2.53 | 2.09 |  | 1.12 |
|  | *h = 5* | 2.41 | 2.02 |  | 1.14 |
|  | *h = 6* | 2.33 | 1.99 |  | 1.31 |
|  | *h = 8* | 3.10 | 2.40 |  | 2.75 |

**The Pseudo Out Of Sample (POOS) Forecasts**

**Period 2**

****

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Model** | **Step-ahead** | **RMSE** | **MAE** | **Correlation** | **Theil's U** |
| *VAR7* | *h = 1* | 1.05 | 0.74 | 1.00 | 8.82 |
|  | *h = 2* | 0.86 | 0.52 | 0.76 | 0.56 |
|  | *h = 3* | 0.82 | 0.56 | 0.75 | 0.55 |
|  | *h = 4* | 0.73 | 0.46 | 0.73 | **0.16** |
|  | *h = 5* | 1.07 | 0.72 | 0.23 | 0.32 |
|  | *h = 6* | 1.01 | 0.69 | 0.40 | 0.32 |
|  | *h = 8* | 0.96 | 0.66 | 0.36 | 0.36 |
| *VAR12* | *h = 1* | 0.65 | 0.46 |  | 5.45 |
|  | *h = 2* | 0.82 | 0.67 |  | 0.60 |
|  | *h = 3* | 0.71 | 0.52 |  | 0.57 |
|  | *h = 4* | 0.82 | 0.65 |  | 4.75 |
|  | *h = 5* | 0.84 | 0.70 |  | 4.70 |
|  | *h = 6* | 0.85 | 0.73 |  | 4.69 |
|  | *h = 8* | 0.77 | 0.65 |  | 4.62 |
| *VAR14* | *h = 1* | 0.04 | 0.03 |  | 0.37 |
|  | *h = 2* | 0.96 | 0.57 |  | 0.74 |
|  | *h = 3* | 1.40 | 0.99 |  | 0.85 |
|  | *h = 4* | 1.40 | 1.07 |  | 5.83 |
|  | *h = 5* | 1.30 | 0.99 |  | 5.77 |
|  | *h = 6* | 1.26 | 1.00 |  | 5.75 |
|  | *h = 8* | 1.54 | 1.14 |  | 5.66 |
| *VAR15* | *h = 1* | 0.91 | 0.64 |  | 7.63 |
|  | *h = 2* | 1.03 | 0.84 |  | 0.73 |
|  | *h = 3* | 0.96 | 0.81 |  | 0.71 |
|  | *h = 4* | 0.86 | 0.67 |  | 0.63 |
|  | *h = 5* | 0.98 | 0.81 |  | 0.65 |
|  | *h = 6* | 1.28 | 1.03 |  | 0.65 |
|  | *h = 8* | 1.17 | 0.93 |  | 0.66 |
| *VAR16* | *h = 1* | 0.29 | 0.21 |  | 2.45 |
|  | *h = 2* | **0.36** | **0.29** |  | **0.26** |
|  | *h = 3* | **0.43** | **0.37** |  | **0.28** |
|  | *h = 4* | **0.52** | **0.45** |  | 3.28 |
|  | *h = 5* | **0.70** | **0.59** |  | 3.25 |
|  | *h = 6* | **0.74** | **0.64** |  | 3.24 |
|  | *h = 8* | **0.70** | **0.60** |  | 3.19 |
| *VAR17* | *h = 1* | 0.52 | 0.37 |  | 4.34 |
|  | *h = 2* | 0.87 | 0.68 |  | 0.65 |
|  | *h = 3* | 0.80 | 0.65 |  | 0.63 |
|  | *h = 4* | 0.72 | 0.53 |  | 0.37 |
|  | *h = 5* | 0.92 | 0.71 |  | 0.42 |
|  | *h = 6* | 0.93 | 0.75 |  | 0.42 |
|  | *h = 8* | 0.88 | 0.70 |  | 0.44 |
| *VAR18* | *h = 1* | 0.74 | 0.52 |  | 6.21 |
|  | *h = 2* | 0.63 | 0.46 |  | 0.42 |
|  | *h = 3* | 0.75 | 0.60 |  | 0.45 |
|  | *h = 4* | 0.71 | 0.59 |  | 2.21 |
|  | *h = 5* | 0.90 | 0.75 |  | 2.19 |
|  | *h = 6* | 0.87 | 0.73 |  | 2.19 |
|  | *h = 8* | 0.93 | 0.78 |  | 2.17 |
| *VAR19* | *h = 1* | 1.05 | 0.74 |  | 8.79 |
|  | *h = 2* | 1.00 | 0.79 |  | 0.69 |
|  | *h = 3* | 0.93 | 0.76 |  | 0.67 |
|  | *h = 4* | 0.85 | 0.69 |  | 1.72 |
|  | *h = 5* | 1.07 | 0.88 |  | 1.72 |
|  | *h = 6* | 1.25 | 1.04 |  | 1.72 |
|  | *h = 8* | 1.15 | 0.96 |  | 1.70 |

**Period 3**

****

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Model** | **Step-ahead** | **RMSE** | **MAE** | **Correlation** | **Theil's U** |
| *VAR7* | *h = 1* | 0.16 | 0.11 | 1.00 | 0.09 |
|  | *h = 2* | 0.52 | 0.36 | 0.88 | 0.69 |
|  | *h = 3* | 1.72 | 1.10 | 0.53 | 0.69 |
|  | *h = 4* | 1.59 | 1.06 | 0.53 | 0.68 |
|  | *h = 5* | 1.46 | 0.94 | 0.53 | 0.68 |
|  | *h = 6* | 1.45 | 1.01 | 0.51 | 0.72 |
|  | *h = 8* | 1.42 | 1.08 | 0.48 | 0.73 |
| *VAR12* | *h = 1* | 0.08 | 0.05 |  | 0.04 |
|  | *h = 2* | 0.21 | 0.15 |  | 0.27 |
|  | *h = 3* | 1.36 | 0.79 |  | 0.41 |
|  | *h = 4* | 1.24 | 0.74 |  | 0.41 |
|  | *h = 5* | 1.15 | 0.69 |  | 0.46 |
|  | *h = 6* | 1.08 | 0.66 |  | 0.41 |
|  | *h = 8* | 1.25 | 0.89 |  | 0.45 |
| *VAR14* | *h = 1* | 0.48 | 0.34 |  | 0.25 |
|  | *h = 2* | 0.82 | 0.64 |  | 1.00 |
|  | *h = 3* | 0.84 | 0.70 |  | 0.77 |
|  | *h = 4* | 1.63 | 1.21 |  | 0.77 |
|  | *h = 5* | 1.66 | 1.31 |  | 1.17 |
|  | *h = 6* | 1.55 | 1.17 |  | 0.98 |
|  | *h = 8* | 1.73 | 1.34 |  | 1.00 |
| *VAR15* | *h = 1* | 0.20 | 0.14 |  | 0.10 |
|  | *h = 2* | 0.90 | 0.60 |  | 1.22 |
|  | *h = 3* | 1.73 | 1.22 |  | 1.02 |
|  | *h = 4* | 1.63 | 1.21 |  | 1.02 |
|  | *h = 5* | 1.49 | 1.04 |  | 0.98 |
|  | *h = 6* | 1.66 | 1.25 |  | 1.11 |
|  | *h = 8* | 1.66 | 1.32 |  | 1.12 |
| *VAR16* | *h = 1* | 0.71 | 0.50 |  | 0.38 |
|  | *h = 2* | **0.64** | **0.49** |  | **0.39** |
|  | *h = 3* | **1.87** | **1.26** |  | **0.56** |
|  | *h = 4* | **1.69** | **1.12** |  | 0.56 |
|  | *h = 5* | **1.57** | **1.04** |  | 0.62 |
|  | *h = 6* | **1.49** | **1.02** |  | 0.59 |
|  | *h = 8* | **1.45** | **1.06** |  | 0.60 |
| *VAR17* | *h = 1* | 0.97 | 0.69 |  | 0.52 |
|  | *h = 2* | 1.02 | 0.83 |  | 0.90 |
|  | *h = 3* | 2.11 | 1.58 |  | 0.85 |
|  | *h = 4* | 1.91 | 1.40 |  | 0.85 |
|  | *h = 5* | 1.75 | 1.24 |  | 0.84 |
|  | *h = 6* | 1.68 | 1.23 |  | 0.78 |
|  | *h = 8* | 1.58 | 1.20 |  | 0.79 |
| *VAR18* | *h = 1* | 0.23 | 0.16 |  | 0.12 |
|  | *h = 2* | 0.72 | 0.51 |  | 0.96 |
|  | *h = 3* | 1.85 | 1.26 |  | 0.87 |
|  | *h = 4* | 1.79 | 1.31 |  | 0.87 |
|  | *h = 5* | 1.64 | 1.13 |  | 0.84 |
|  | *h = 6* | 1.60 | 1.16 |  | 0.81 |
|  | *h = 8* | 1.47 | 1.08 |  | 0.82 |
| *VAR19* | *h = 1* | 0.34 | 0.24 |  | 0.18 |
|  | *h = 2* | 0.69 | 0.53 |  | 0.88 |
|  | *h = 3* | 1.67 | 1.17 |  | 0.79 |
|  | *h = 4* | 1.59 | 1.18 |  | 0.78 |
|  | *h = 5* | 1.45 | 1.01 |  | 0.75 |
|  | *h = 6* | 1.50 | 1.12 |  | 0.83 |
|  | *h = 8* | 1.57 | 1.25 |  | 0.85 |

**Overall three periods**

Best overall: VAR7 & VAR17

Second bests: VAR12 (performs very well in terms of the traditional measures RMSE and MAE)

Third Best: VAR 16 VAR 18

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Model** | **Step-ahead** | **RMSE** | **MAE** | **Correlation** | **Theil's U** |
| *VAR7* | *h = 1* | 0.51 | 0.36 | 1.00 | 2.97 |
|  | *h = 2* | 0.74 | 0.50 | 0.87 | 0.44 |
|  | *h = 3* | 1.10 | 0.73 | 0.75 | 0.44 |
|  | *h = 4* | 1.38 | 1.11 | 0.72 | 0.42 |
|  | *h = 5* | 1.51 | 1.03 | 0.56 | 0.53 |
|  | *h = 6* | 1.44 | 1.00 | 0.61 | 0.55 |
|  | *h = 8* | 1.64 | 1.21 | 0.51 | 0.82 |
| *VAR12* | *h = 1* | 0.91 | 0.64 |  | 1.86 |
|  | *h = 2* | 0.90 | 0.65 |  | 0.32 |
|  | *h = 3* | 1.17 | **0.72** |  | 0.36 |
|  | *h = 4* | **1.14** | **0.75** |  | 1.76 |
|  | *h = 5* | **1.07** | **0.71** |  | 1.76 |
|  | *h = 6* | **1.07** | **0.74** |  | 1.89 |
|  | *h = 8* | **1.13** | **0.85** |  | 1.99 |
| *VAR14* | *h = 1* | 0.47 | 0.33 |  | 0.22 |
|  | *h = 2* | **1.82** | 1.24 |  | 0.68 |
|  | *h = 3* | 1.83 | 1.28 |  | 0.67 |
|  | *h = 4* | **2.20** | **1.64** |  | **2.42** |
|  | *h = 5* | **2.12** | **1.64** |  | **2.56** |
|  | *h = 6* | 2.00 | 1.53 |  | **2.55** |
|  | *h = 8* | **2.61** | **1.90** |  | **3.52** |
| *VAR15* | *h = 1* | 1.34 | 0.94 |  | 2.62 |
|  | *h = 2* | 1.67 | **1.31** |  | **0.72** |
|  | *h = 3* | **2.19** | **1.77** |  | **1.08** |
|  | *h = 4* | 2.02 | 1.62 |  | 1.05 |
|  | *h = 5* | 1.92 | 1.51 |  | 1.05 |
|  | *h = 6* | **2.01** | 1.59 |  | 1.11 |
|  | *h = 8* | 2.13 | 1.74 |  | 1.47 |
| *VAR16* | *h = 1* | 1.14 | 0.81 |  | 0.98 |
|  | *h = 2* | 0.99 | 0.68 |  | **0.25** |
|  | *h = 3* | 1.34 | 0.88 |  | **0.32** |
|  | *h = 4* | 1.27 | 0.86 |  | 1.33 |
|  | *h = 5* | 1.25 | 0.85 |  | 1.35 |
|  | *h = 6* | 1.23 | 0.88 |  | 1.46 |
|  | *h = 8* | 1.23 | 0.94 |  | 1.56 |
| *VAR17* | *h = 1* | 1.43 | 1.01 |  | 1.66 |
|  | *h = 2* | 1.40 | 1.00 |  | 0.56 |
|  | *h = 3* | 1.64 | 1.14 |  | 0.54 |
|  | *h = 4* | 1.48 | 1.00 |  | 0.45 |
|  | *h = 5* | 1.46 | 1.01 |  | **0.49** |
|  | *h = 6* | 1.41 | 1.02 |  | **0.55** |
|  | *h = 8* | 1.34 | 1.01 |  | **0.64** |
| *VAR18* | *h = 1* | 0.59 | 0.42 |  | 2.12 |
|  | *h = 2* | 0.72 | 0.54 |  | 0.48 |
|  | *h = 3* | 1.10 | 0.79 |  | 0.46 |
|  | *h = 4* | 1.33 | 0.97 |  | 1.14 |
|  | *h = 5* | 1.37 | 1.02 |  | 1.16 |
|  | *h = 6* | 1.31 | 0.98 |  | 1.16 |
|  | *h = 8* | 1.51 | 1.15 |  | 1.38 |
| *VAR19* | *h = 1* | 1.12 | 0.79 |  | 3.02 |
|  | *h = 2* | 1.25 | 0.99 |  | 0.57 |
|  | *h = 3* | 1.78 | 1.41 |  | 0.85 |
|  | *h = 4* | 1.66 | 1.32 |  | 1.21 |
|  | *h = 5* | 1.64 | 1.30 |  | 1.20 |
|  | *h = 6* | 1.69 | 1.38 |  | 1.29 |
|  | *h = 8* | 1.94 | 1.54 |  | 1.76 |